

# Storing light with correlations in arrays of atoms

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We show how strong light-mediated resonant dipole-dipole interactions between atoms can be utilized in a control and storage of light. The method is based on a high-fidelity preparation of a collective atomic excitation in a single correlated subradiant eigenmode in a lattice. We demonstrate how a simple phenomenological model captures the qualitative features of the dynamics and scattering resonances.

Resonant emitters play a key role in optical devices for classical and quantum technologies. Atoms have particular advantages because of an excellent isolation from environmental noise with well-specified resonance frequencies and no absorption where a photon is lost. At high densities, however, they exhibit strong light-mediated resonant dipole-dipole (DD) interactions that can lead to uncontrolled and unwanted phenomena, such as resonance broadening, shifts and dephasing. According to common wisdom, these are considered as a design limitation in quantum technologies, e.g., in quantum metrology [1, 2], sensing [3], information processing [4], in the storage of light and in the implementations of quantum memories [5–8]. Here we show how strong radiative interactions can be harnessed in engineering long-living *collective* excitations that may be utilized in the control and storage of light. Our protocol is based on controlled preparation of large, many-atom subradiant excitations, where the light-mediated interactions between the atoms strongly suppress any radiative losses.

Superradiance [9] where the emission of light is coherently enhanced in an ensemble of emitters has continued to attract considerable interest [10] with the recent experiments focusing on light in confined geometries [11], weak excitation regime [12–14], and the related shifts of the resonance frequencies [15–19]. Its counterpart, subradiance, describes coherently suppressed emission due to a weak coupling to the radiative vacuum. Because of the weak coupling, subradiant states are challenging to excite and have experimentally proved elusive. In atomic and molecular systems subradiance has been observed in pairs of trapped ions [20] and molecules [21], as well as in weakly bound ultracold molecular states [22, 23]. In a large atom cloud a subradiant decay was recently observed in the long tails of a radiative decay distribution [24] that indicated a small fraction of the atoms exhibiting a suppressed emission.

In our model, an incident light excites a collective atomic state that exhibits a significant radiative vacuum coupling. The excitation is then transferred to a radiatively isolated cooperative state. The cold atoms that store the light excitation are confined in a planar lattice, providing a protection against nonradiative losses—that

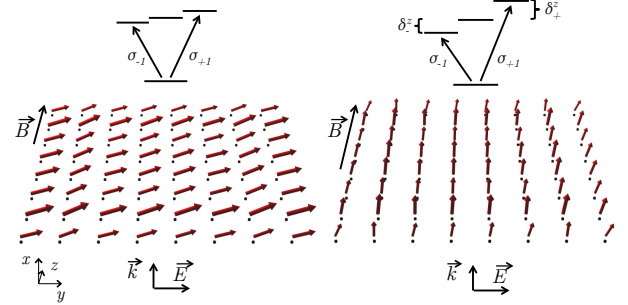


FIG. 1. Schematic illustration and numerically calculated response. The atoms (one per site) are confined in a small square 2D  $8 \times 8$  array on the  $yz$  plane. The linearly polarized (along  $y$ ) incident light propagates along the positive  $x$  direction driving the  $|J = 0, m_J = 0\rangle \rightarrow |J' = 1, m_J = \pm 1\rangle$  transitions. The arrows represent the numerically calculated steady-state atomic dipoles at each site. A real or synthetic magnetic field along the  $z$  axis induces Zeeman shifts, effectively rotating the dipoles around the  $z$  axis. For  $(\delta_z^+, \delta_z^-) = (0.1, 0.3)\gamma$  (left panel) this rotation is small, but for  $(\delta_z^+, \delta_z^-) = (0.45, 1.75)\gamma$  (driven at the resonance of the subradiant mode; right panel), the dipoles are oriented approximately normal to the lattice, representing a collective excitation eigenmode with a factor of 50 narrowed linewidth.

typically are a common hindrance to observation of subradiance. The state transfer is achieved by rotating the collective atomic polarization by an effective magnetic field. Depending on the size of the lattice and the confinement of the atoms, we find substantially suppressed radiative emission where up to 98-99% of the total excitation is transferred into a *single* subradiant eigenmode of the interacting multiatom system. The correlated many-atom excitation spatially extends over the entire lattice and is therefore fundamentally different from two-atom subradiant states [20, 22, 23]. We develop a simple phenomenological two-mode model that provides an intuitive description of the light storage dynamics, and qualitatively captures the essential features, e.g., of the Fano resonance of the forward-scattered light.

We consider a tightly-confined square planar array of atoms (e.g., a 2D optical lattice) with one atom per site (Fig. 1). The light-induced radiative DD interactions lead to collective behavior of the atoms in the lattice that is dramatically different from the response of an in-

dividual, isolated atom [25–29]. The atoms are either at fixed positions or we address the position fluctuations using the model of a finite optical lattice with the potential depth  $sE_R$  in the units of the lattice photon recoil energy  $E_R$  [30] (Appendix). In the numerics, the lattice spacing  $a = 0.55\lambda$ , except when specified otherwise. Whenever we consider a finite lattice depth, we take the confinement normal to the lattice  $\simeq 0.12a$ . The atoms are illuminated by an incident weak-intensity laser with the amplitude  $\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(y, z)\hat{\mathbf{e}}_y \exp(ikx)$ , with polarization  $\hat{\mathbf{e}}_y$  and  $\mathcal{E}_0(y, z)$  either constant or a Gaussian profile on the  $yz$  plane. Here, and in the rest of the paper, all the field amplitudes and the atomic polarization correspond to the slowly varying positive frequency components with oscillations at the laser frequency  $\omega$ . We consider a near-resonance  $J = 0 \rightarrow J' = 1$  atomic transition and assume a controllable Zeeman level splitting of the  $J' = 1$  manifold. The Zeeman shifts could be induced by magnetic fields or, e.g., by AC Stark shifts [31].

In the numerical simulations we calculate the optical response by evaluating all the multiple scattering events [32, 33] between the atoms in an array. In the limit of low light intensity, for stationary atoms the results are exact [34, 35], and we also include the vacuum fluctuations of the atomic positions in the lowest vibrational level of each lattice site [25]. This is done by stochastically sampling the atomic positions at each site in each realization according to the density distribution and then ensemble-averaging the results. At each stochastic run we have the  $N$  atoms fixed at positions  $\mathbf{r}_j$ , and we calculate the dipole moment  $\mathbf{d}_j = \mathcal{D} \sum_{\sigma} \hat{\mathbf{e}}_{\sigma} \mathcal{P}_{\sigma}^{(j)}$  for each atom  $j$ , where  $\mathcal{D}$  denotes the reduced dipole matrix element. Each atom has three polarization amplitude components  $\mathcal{P}_{\sigma}^{(j)}$  associated with the unit circular polarization vectors  $\hat{\mathbf{e}}_{\pm 1} = \mp(\hat{\mathbf{e}}_x \pm i\hat{\mathbf{e}}_y)/\sqrt{2}$  and  $\hat{\mathbf{e}}_0 = \hat{\mathbf{e}}_z$ , that are coupled with the transitions  $|J = 0, m = 0\rangle \rightarrow |J' = 1, m = \sigma\rangle$ .

In the limit of low light intensity, the excited state population of the atoms vanishes and the excitation amplitudes satisfy [25, 35]

$$\frac{d}{dt} \mathcal{P}_{\sigma}^{(j)} = (i\Delta_{\sigma} - \gamma) \mathcal{P}_{\sigma}^{(j)} + i\frac{\xi}{\mathcal{D}} \hat{\mathbf{e}}_{\sigma}^* \cdot \epsilon_0 \mathbf{E}_{\text{ext}}(\mathbf{r}_j), \quad (1)$$

where  $\xi = 6\pi\gamma/k^3$  and the single-atom Wigner-Weisskopf linewidth  $\gamma = \mathcal{D}^2 k^3 / (6\pi\hbar\epsilon_0)$ . The detuning from the atomic resonance  $\Delta_{\sigma} = \omega - \omega_{\sigma} = \omega - (\omega_0 + \sigma\delta_{\sigma}^z)$  where  $\omega_0$  is the resonance frequency of the  $|J = 0\rangle \leftrightarrow |J' = 1, m = 0\rangle$  transition and  $\pm\delta_{\pm}^z$  are the shifts of the  $m = \pm 1$  levels (Fig. 1). Each amplitude in Eq. (1) is driven by the sum of the incident field and the fields scattered from all the other  $N - 1$  atoms  $\mathbf{E}_{\text{ext}}(\mathbf{r}_j) = \mathcal{E}(\mathbf{r}_j) + \sum_{l \neq j} \mathbf{E}_S^{(l)}(\mathbf{r}_j)$ . The scattered dipole radiation field from the atom  $l$  is  $\epsilon_0 \mathbf{E}_S^{(l)}(\mathbf{r}) = \mathbf{G}(\mathbf{r} - \mathbf{r}_l) \mathcal{D} \sum_{\sigma} \hat{\mathbf{e}}_{\sigma} \mathcal{P}_{\sigma}^{(l)}$ , where  $\mathbf{G}$  is the dipole radiation kernel, such that  $\mathbf{E}_S^{(l)}(\mathbf{r})$  represents the electric field at  $\mathbf{r}$  from a dipole  $\mathcal{D} \sum_{\sigma} \hat{\mathbf{e}}_{\sigma} \mathcal{P}_{\sigma}^{(l)}$  residing at  $\mathbf{r}_l$  [36].

We first consider a single, isolated atom. This is ob-

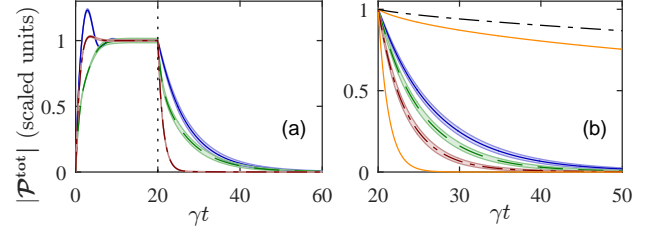


FIG. 2. The dynamics of the total atomic polarization density in the lattice of  $20 \times 20$  sites for different Zeeman shifts and lattice heights. (a)  $(\delta_+^z, \delta_-^z, \Delta_0) = (0, 0, 0)$  (dashed-dotted, red curve),  $(0.4, 0.6, 0)\gamma$  (dashed, green curve), and  $(1.1, 1.1, 0.65)\gamma$  (solid, blue curve) for  $s = 50$ . At  $\gamma t = 20$  (when each curve is normalized to one), the Zeeman shifts and the incident light are switched off. When the dipoles are oriented close to the  $x$  axis (see Fig. 1 on right), the decay is slow (collective sub-radiance). For  $\delta_{\pm}^z = 0$  the dipoles are pointing along the  $y$  axis and decay rapidly. An exponential fitting provides decay rates  $0.79\gamma$ ,  $0.16\gamma$ , and  $0.14\gamma$ . (b) The curves from top to bottom: incident Gaussian beam with fixed atomic positions, plane-wave excitation for fixed atomic positions, for lattice with  $s = 50, 20, 5$ ,  $[(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma]$ , and for fixed atomic positions  $(\delta_{\pm}^z = \Delta_0 = 0)$ . In (a) and (b) the shaded lines around the curves represent the stochastic uncertainties of the excitation amplitudes due to the vacuum fluctuations of the atomic positions.

tained in Eq. (1) by setting  $\mathbf{E}_{\text{ext}}(\mathbf{r}_j) \rightarrow \mathcal{E}(\mathbf{r}_j)$ . The  $y$ -polarized light then drives the atomic polarization components  $\mathcal{P}_{\pm 1}^{(j)}$  (Fig. 1). Here we instead write the equations of motion in the Cartesian basis  $\mathbf{d}_j/\mathcal{D} = \hat{\mathbf{e}}_x \mathcal{P}_x^{(j)} + \hat{\mathbf{e}}_y \mathcal{P}_y^{(j)} + \hat{\mathbf{e}}_z \mathcal{P}_z^{(j)}$ , such that  $\mathcal{P}_x^{(j)} = (\mathcal{P}_{-1}^{(j)} - \mathcal{P}_{+1}^{(j)})/\sqrt{2}$  and  $\mathcal{P}_y^{(j)} = -i(\mathcal{P}_{-1}^{(j)} + \mathcal{P}_{+1}^{(j)})/\sqrt{2}$ . We obtain

$$\dot{\mathcal{P}}_x^{(j)} = (i\Delta_0 - i\tilde{\delta} - \gamma)\mathcal{P}_x^{(j)} - \bar{\delta}\mathcal{P}_y^{(j)}, \quad (2)$$

$$\dot{\mathcal{P}}_y^{(j)} = (i\Delta_0 - i\tilde{\delta} - \gamma)\mathcal{P}_y^{(j)} + \bar{\delta}\mathcal{P}_x^{(j)} + i\xi\epsilon_0\mathcal{E}_0/\mathcal{D}, \quad (3)$$

where  $\tilde{\delta} = (\delta_+^z - \delta_-^z)/2$ ,  $\bar{\delta} = (\delta_+^z + \delta_-^z)/2$ , and  $\Delta_0$  denotes the detuning of the  $m = 0$  state. The incident light directly drives only  $\mathcal{P}_y^{(j)}$ , but the energy splitting of the levels  $|m = \pm 1\rangle$  introduces a coupling between  $\mathcal{P}_x^{(j)}$  and  $\mathcal{P}_y^{(j)}$ . Although the incident field is perpendicular to  $\mathcal{P}_x^{(j)}$ , the light can therefore still excite  $\mathcal{P}_x^{(j)}$  by first driving  $\mathcal{P}_y^{(j)}$ . The  $J = 0 \rightarrow J' = 1$  transition is isotropic when the excited-state energies are degenerate and any orientation of the orthogonal basis also forms an eigenbasis. For  $\bar{\delta} \neq 0$ ,  $\mathcal{P}_{x/y}^{(j)}$  no longer are eigenstates. The dipoles are consequently turned toward the  $x$  axis by the rotation around the effective magnetic field.

For the entire interacting many-body system we numerically calculate the optical response for different Zeeman shifts and lattice heights and show in Fig. 2(a) the dynamics of the total polarization of the system  $|\mathcal{P}^{\text{tot}}| = |\sum_{j,k} \mathcal{P}_k^{(j)} \hat{\mathbf{e}}_k|/N$  [in all the numerical results, the polarization amplitudes are expressed in the dimensionless form  $\mathcal{P} \rightarrow \mathcal{D} \mathcal{P} k^3 / (6\pi\epsilon_0\mathcal{E}_0)$ ]. The incident light

excites the  $y$  components of the atomic dipoles. Analogously to the single atom case, the Zeeman shifts turn the polarization density toward the  $x$  direction. At the resonance (Appendix)  $(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma$  we find the dipoles almost entirely along the  $x$  direction [37]. After the evolution has reached the steady state, the Zeeman shifts and the incident laser are turned off, resulting in a decay of the excitations. We fit the exponential functions to the decay profiles to obtain numerical estimates for the collective radiative linewidths that we later compare with the collective eigenvalues. For  $\delta_\pm^z = 0$ , the dipoles are in the lattice plane and the radiative decay rate  $0.79\gamma$  is close to the single atom linewidth. However, for  $(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma$  we find strongly suppressed decay of  $0.14\gamma$ , indicating that the *entire* collective radiative excitation is dominated by subradiance. This is very different from the observation of long tails of radiative decay where only an extremely small fraction of the total excitation exhibits enhanced lifetime [24].

The lattice confinement affects the subradiant decay [Fig. 2(b)] and for more strongly fluctuating atomic positions we obtain faster decay rates with  $0.18\gamma$  and  $0.28\gamma$  for  $s = 20$  and  $s = 5$ , respectively. For the case of fixed atomic positions a better fit is obtained by a double exponential (reflecting the occupation of eigenmodes with different linewidths, as explained later)  $b_1 e^{-c_1 t} + b_2 e^{-c_2 t}$ , with  $b_1 \simeq 0.72$ ,  $c_1 \simeq 0.0032\gamma$ ,  $b_2 \simeq 0.24$ ,  $c_2 \simeq 0.027\gamma$ . The decay is dominated by an exponent that is about 300 times smaller than the one for a single atom.

Owing to the resonant DD interactions the atoms respond collectively to light, exhibiting collective excitation eigenmodes with distinct collective radiative linewidths and line shifts. We can qualitatively understand the response by analyzing the behavior of the most dominant modes. The incident light is phase-matched to a smoothly-varying, phase-coherent excitation of the atoms. The linear polarization couples to a collective mode in which all the dipoles are coherently oscillating along the  $y$  direction with the excitation  $\mathcal{P}_I$ . Such a mode closely approximates a collective eigenmode of the system in the absence of the Zeeman shifts. Since all the dipoles in this mode are in the lattice plane,  $\mathcal{P}_I$  is responsible for strong reflection and transmission of light. For nonzero Zeeman shifts the mode no longer is an eigenmode, but as in the single atom case, the polarization of the atoms is then turned toward the  $x$  axis. This reorientation can be qualitatively analyzed by a simple two-mode model when we assume that  $\mathcal{P}_I$  is predominantly coupled with a phase-coherent collective excitation  $\mathcal{P}_P$  where all the atomic dipoles are oscillating in phase, normal to the plane (Appendix). Also this mode is a collective eigenmode for  $\delta_\pm^z = 0$ . We can now establish an effective two-mode dynamics (Appendix)

$$\dot{\mathcal{P}}_P = (i\Delta_P - i\tilde{\delta} - v_P)\mathcal{P}_P - \bar{\delta}\mathcal{P}_I, \quad (4a)$$

$$\dot{\mathcal{P}}_I = (i\Delta_I - i\tilde{\delta} - v_I)\mathcal{P}_I + \bar{\delta}\mathcal{P}_P + i\xi\epsilon_0\mathcal{E}_0/\mathcal{D}, \quad (4b)$$

where  $v_{P/I}$  are the collective linewidths of the corresponding eigenmodes of the many-atom system (for  $\delta_\pm^z = 0$ ) and  $\Delta_{P/I} = \omega - \omega_{P/I} = \Delta_0 + \delta_{P/I}$  are the detunings of the incident light from the resonances of these modes (that are shifted by  $\delta_{P/I}$ ).

The excitation  $\mathcal{P}_P$  dominantly radiates within the plane, resulting in enhanced interactions between the atoms: For light to escape the system it generally undergoes many scattering events in large lattices, so that the collective mode becomes strongly subradiant.

We also calculate the eigenmodes when  $\delta_\pm^z = 0$  for the full interacting system of atoms and light, and analyze the occupations of the different eigenmodes in the steady-state responses of Fig. 2(b) (at  $\gamma t = 20$ ). We use the occupation measure  $L_j = |\mathbf{v}_j^T \mathbf{b}|^2 / \sum_i |\mathbf{v}_i^T \mathbf{b}|^2$  for the eigenvector  $\mathbf{v}_j$  in the state  $\mathbf{b}$ . The resonance linewidths are then compared with the calculated decay rates of the excitations in Fig. 2(b). We find that the steady-state excitation of the  $\delta_\pm^z = 0$  fixed atomic position case is dominated by the collective  $\mathcal{P}_I$  excitation eigenmode with about 50% of the total excitation (Appendix). Its linewidth  $v_I \simeq 0.79\gamma$  almost perfectly matches with the fitted decay rate  $0.80\gamma$  in Fig. 2(b). For the  $(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma$  fixed atomic position case the fitting of the radiative decay to a double-exponential in Fig. 2(b) provided a much better result. This slowly-decaying case is dominated by the subradiant  $\mathcal{P}_P$  excitation eigenmode with about 70% of the total excitation (Appendix). The linewidth  $v_P \simeq 3.1 \times 10^{-3}\gamma$  indicates a strongly subradiant excitation and again very closely matches with the dominant exponent  $3.2 \times 10^{-3}\gamma$  of the decay in Fig. 2(b). The reason for the double-exponential decay in Fig. 2(b) is a prominent excitation  $\sim 15\%$  of an additional eigenmode whose linewidth  $\simeq 0.015\gamma$  notably differs from that of  $\mathcal{P}_P$ .

Although the subradiant eigenmode with  $v_P \simeq 3.1 \times 10^{-3}\gamma$  has a uniform phase profile, its amplitude is smaller close to the lattice edges (Appendix). This suggests that even a more targeted excitation of this mode can be achieved using a focused Gaussian laser beam. Indeed, a Gaussian beam with the standard deviation  $6a$  increases the occupation to 98% of the total excitation (Appendix). The corresponding dynamics provides an excellent fit to a *single* exponential with a decay rate of  $3.1 \times 10^{-3}\gamma$ .

The many-body nature of the light-mediated interactions manifests itself in a strong dependence of the suppressed decay on the size of the system. In Fig. 3(b) we show the linewidth  $v_P$  as a function of the number of atoms  $N$ . For fixed atomic positions the mode becomes increasingly more subradiant in larger lattices with  $v_P/\gamma \simeq N^{-0.91}$ . The fluctuations of the atomic positions suppress the linewidth narrowing and, e.g., at  $s = 50$  approaches an asymptotic value  $v_P \simeq 0.15\gamma$  in the large lattice limit. Provided that the atoms could be confined sufficiently tightly, e.g., by optical tweezers, the

lifetime of the collective light storage can be significantly increased in large systems.

By varying the lattice spacing for different atom numbers we find that  $v_P$  has a minimum around  $a/\lambda=0.7-0.8$  [Fig. 3(a)]. Around the minimum  $v_P$  is also the most subradiant linewidth of the system. The engineered excitations have particularly narrow linewidths for far red-detuned optical lattices for which  $a/\lambda \gtrsim 0.55$ .

The narrow linewidth  $v_P$  manifests itself also in the resonance of the scattered light (Fig. 4). We display the spectrum of the steady-state response of the forward scattered light into a narrow cone of  $|\sin \theta| \lesssim 0.1$ . The full numerical simulation is compared with the two-mode model of Eqs. (4) that qualitatively captures the main features of the spectra, indicating that the resonance behavior is dominated by the two collective modes. The spectra exhibit a Fano resonance due to a destructive interference between different scattering paths that involve either the excitation  $\mathcal{P}_I$  only, or a scattering via  $\mathcal{P}_P$ , as in  $\mathcal{P}_I \rightarrow \mathcal{P}_P \rightarrow \mathcal{P}_I$ . One can see from Eqs. (4) (Appendix) that the forward or back scattered light is suppressed when  $\bar{\delta}^2 \gg v_P v_I$  and that the resonances correspond to high (low) occupations of  $\mathcal{P}_P$  ( $\mathcal{P}_I$ ) excitations. In the limit that  $\mathcal{P}_P$  is not strongly driven, the narrow spectral resonance is a direct consequence of its subradiant linewidth in a large lattice (the resonances strongly depend on the lattice size; Fig. 4), and the interference is analogous to the interference of bright and dark modes in the electromagnetically-induced transparency (EIT) [38]. If  $\mathcal{P}_P$  is strongly excited by the Zeeman shifts, the resonance notably broadens and its width can be approximated by  $[(v_I^2 + 4\bar{\delta}^2)^{1/2} - v_I]/2$  (for  $v_P/\bar{\delta} \simeq 0$ ) (Appendix). Interestingly, in the limit of a large lattice the optical response can be shown to vary between a full transmission ( $\mathcal{P}_P$  resonance) and complete reflection (for zero Zeeman shifts) (Appendix). Narrow transmission resonances due to collective radiative interference may also be achieved in magnetodielectric solid-state resonator systems [39], and EIT in an optical lattice has been proposed [27].

In conclusion, we showed that collective many-atom interactions can be beneficial for quantum technologies with applications in a controlled preparation of subradiant light storage. Almost all the light can be stored in a single atomic excitation eigenmode of the interacting light-matter system that extends over the entire lattice. The process is reversible and light can be released by ramping down the magnetic field. The possibility to engineer optical interactions may be promising, e.g., for the control of many-atom light shifts in lattice clocks [1, 2], and our subradiant state also exhibits suppressed shifts (Appendix). Moreover, the narrow resonance features are very sensitive to the Zeeman shifts and could potentially also provide a sensitive detection mechanism of weak magnetic fields. Unlike in a magnetometry using EIT [40] in weakly interacting vapors, the resonance here

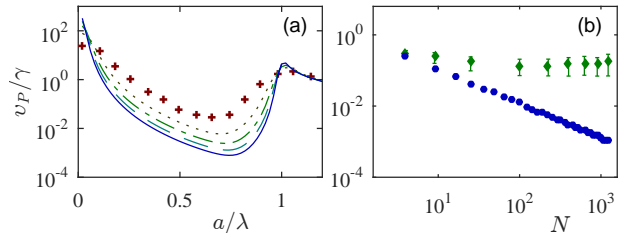


FIG. 3. The resonance linewidth  $v_P$  in the units of a single atom linewidth  $\gamma$  of the subradiant collective eigenmode where the atomic dipoles coherently point to the normal of the lattice. (a) The dependence on the lattice spacing  $a$ . The curves from top to bottom: lattices  $5 \times 5$ ,  $10 \times 10$ ,  $15 \times 15$ ,  $20 \times 20$ , and  $25 \times 25$ . (b) The dependence on the number of atoms for  $a = 0.55\lambda$ :  $s = 50$  (top curve), the atoms at fixed positions (lower curve). For instance,  $v_P \simeq 1.0 \times 10^{-3}\gamma$  for the  $35 \times 35$  lattice in the lower curve. In comparison, the most subradiant eigenmode in this case has a linewidth  $1.5 \times 10^{-4}\gamma$ .

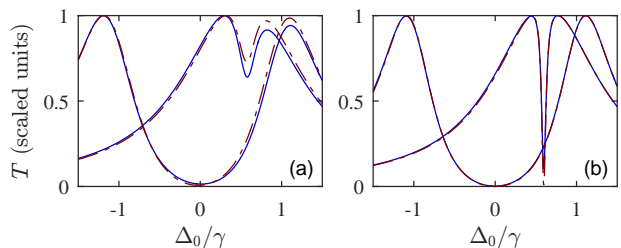


FIG. 4. The spectrum of forward or back scattered light and Fano resonances for different orientations of the dipoles:  $(\delta_+^z, \delta_-^z) = (0.1, 0.2)\gamma$  (the orientation not far from the lattice plane; narrow resonances),  $(0.45, 1.75)\gamma$  (the orientation approximately normal to the plane; broad resonances) for a lattice of fixed atomic positions and (a)  $3 \times 3$ ; (b)  $20 \times 20$  sites. Numerical simulation (blue, solid curves), two-mode model (red, dashed curves) with numerically calculated eigenvalues for the two dominant eigenmodes  $(\delta_P + iv_P)/\gamma \simeq -0.65 + 0.0031i$  ( $20 \times 20$ ),  $-0.62 + 0.12i$  ( $3 \times 3$ ), and  $(\delta_I + iv_I)/\gamma \simeq -0.68 + 0.79i$  ( $20 \times 20$ ),  $-0.59 + 0.83i$  ( $3 \times 3$ ). The scattering resonance is approximately at the effective resonance of the subradiant mode [Eq. (4a)]  $\Delta_P - \bar{\delta} \simeq 0$ .

is not limited by the single atom linewidth, but by the much narrower subradiant cooperative linewidth, resulting, e.g., in a high sensitivity of the effective refractive index to the level shifts.

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## Appendices

### Array of atoms

We assume the atoms to be in a planar square array. We label the sites of the  $N_y \times N_z$  lattice with the



lattice spacing  $a$  as  $i = 1, \dots, N$ ,  $N = N_y N_z$ . The array could be formed by optical tweezers, optical lattice potential, or by nanofabrication. In the case of an optical lattice we approximate the lattice site Wannier function  $\phi_i(\mathbf{r}) \equiv \phi(\mathbf{r} - \mathbf{R}_i)$ , centered at  $\mathbf{R}_i$ , by the ground-state wavefunction of a harmonic oscillator, with the vibrational frequency  $\omega = 2\sqrt{s}E_R/\hbar$  and the  $1/e$  radius  $\ell = \ell_y = \ell_z = as^{-1/4}/\pi$  of the atom density  $\rho_i(\mathbf{r}) \equiv |\phi_i(\mathbf{r})|^2$ , where  $E_R = \pi^2\hbar^2/(2ma^2)$  is the lattice-photon recoil energy and each site has a potential depth  $sE_R$  [30]. In the  $yz$  plane the atoms become more localized as the lattice depth  $s$  is increased. We assume that the confinement of the atoms around the  $x = 0$  plane results in  $1/e$  radius of the atom density equal to  $\ell_x$ . In this work, whenever we consider fluctuating atomic positions, we take  $\ell_x \simeq 0.12a$ .

We assume one atom per site. In an optical lattice precise occupation numbers can be achieved by using ultracold atoms in a Mott-insulator state [30]. Experimentally, the atoms in the Mott state can be prepared to a single-atom site-occupancy even in situations where the lattice is superposed with a harmonic trapping potential by manipulating the sites with excess occupancy [41].

### Calculation of eigenmodes

The collective radiative excitation eigenmodes of the full system of  $N$  atoms can be solved writing the coupled system of atoms and light [Eq. (1) in the main text] as  $\dot{\mathbf{b}} = i\mathcal{H}\mathbf{b} + \mathbf{F}$ , where  $\mathbf{b}$  is a vector made of the amplitudes  $\mathcal{P}_k^{(j)}$  and  $\mathbf{F}$  represents the external driving of the dipoles by the incident light [?]. The coupling matrix  $i\mathcal{H}$  provides the light-induced interactions between the atoms and the first terms on the right-hand-side of Eq. (1) in the main text. The matrix  $\mathcal{H}$  has  $3N$  eigenmodes  $\mathbf{v}_j$  with the eigenvalues  $\delta_j + iv_j$  where  $\delta_j = \omega_0 - \omega_j$  is the shift of the collective mode resonance  $\omega_j$  from the single atom resonance and  $v_j$  is the collective radiative linewidth.

In Fig. A1 we show the probability distributions of the collective eigenmode resonance linewidths. The increase in the size of the lattice results in a larger number of subradiant eigenmodes with notably narrower linewidths. The collective subradiant eigenmode where the atomic dipoles are coherently oscillating in the  $x$  direction is displayed in Fig. A2; the distribution of the complex amplitudes demonstrates the phase coherence over the entire lattice.

Although the eigenvectors here form a basis, they are generally not orthogonal, since  $\mathcal{H}$  is not Hermitian. For the isotropic  $J = 0 \rightarrow J' = 1$  system  $\mathcal{H}$  is symmetric, and we can determine the biorthogonality condition  $\mathbf{v}_j^T \mathbf{v}_i = \delta_{ji}$ , except for possible zero-binorm states for which  $\mathbf{v}_j^T \mathbf{v}_j = 0$  (that we have not encountered in our system). We therefore define an overlap measure  $L_j = |\mathbf{v}_j^T \mathbf{b}|^2 / \sum_i |\mathbf{v}_i^T \mathbf{b}|^2$  for the eigenvector  $\mathbf{v}_j$  in the state

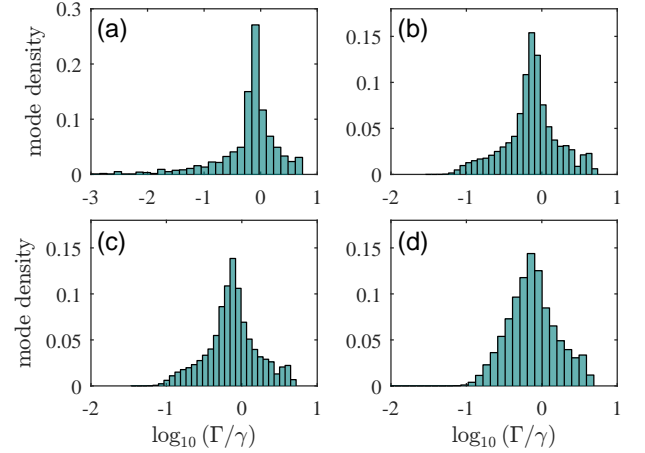


FIG. A1. Probability distributions (normalized to one) of collective radiative eigenmode resonance linewidths for (a) fixed atomic positions, (b) the optical lattice height  $s = 50$ , (c)  $s = 20$ , (d)  $s = 5$ . The distributions become narrower as the atoms are less tightly confined. This is particularly manifested by the disappearance of the long tails of strongly subradiant eigenmode decay rates in the distribution as the lattice becomes shallower.

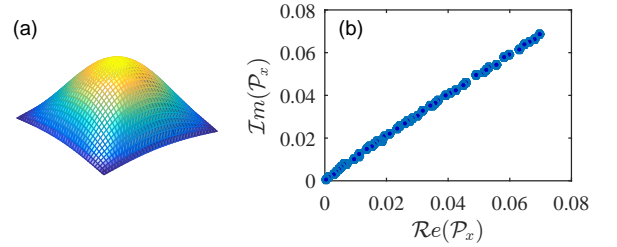


FIG. A2. The collective subradiant eigenmode where the atomic dipoles are coherently oscillating in the  $x$  direction for the case of fixed atomic positions. (a) The absolute value of the amplitude of the  $x$ -component of the atomic polarization density on the lattice plane. The excitation is largest at the center of the lattice and decreases toward the edges. The corresponding  $y$  and  $z$  components are approximately zero. (b) The  $x$ -component of the complex polarization density for each lattice component that exhibits high phase coherence over the entire lattice.

b. We also use this measure to determine the eigenmodes that are the closest to the ideal phase-coherent polarization excitations  $\mathcal{P}_I$  and  $\mathcal{P}_P$ .

In Fig. A3 we show the populations of the different eigenmodes in the steady-state response for the case where the subradiant mode with the polarization vectors normal to the lattice plane is excited and the case where it is not. In all the cases only a small number of modes are significantly excited. In the case of driving of the dipole excitation normal to the lattice using a plane wave the excited eigenmodes have different linewidths resulting in an approximate double-exponential decay rate that is described in the main text. For the Gaussian beam driving, only one eigenmode is notably excited. This is because the Gaussian beam intensity is better matched with the density distribution of the eigenmode in Fig. A2.

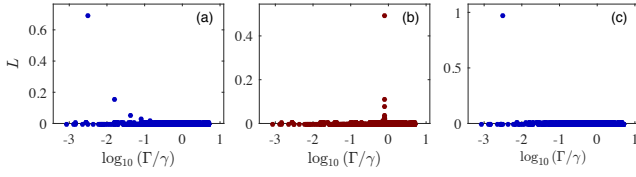


FIG. A3. The measure of the mode population  $L$  of the different eigenmodes in the steady-state response for fixed atomic positions, (a) incident plane wave excitation,  $(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma$ ; (b) incident plane wave excitation,  $(\delta_+^z, \delta_-^z, \Delta_0) = (0, 0, 0)\gamma$ ; (c) Gaussian beam excitation with the standard deviation  $6a$ ,  $(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma$ . The eigenmodes are ordered by their collective radiative resonance linewidths on a logarithmic scale. In (b) the atomic polarization density is in the lattice plane and the excitation is dominated (over 50% of the total excitation) by the collective mode where the dipoles are coherently oscillating in the  $y$  direction. In (a) and (c) the atomic polarization density is pointing normal to the lattice plane, and the excitation is dominated by about (a) 70%, (c) 98% by the collective subradiant mode where the atomic dipoles are coherently oscillating in the  $x$  direction. In (a) also some modes with notably broader resonances are occupied that manifests themselves in a faster initial decay of the radiative excitation (see Fig. 2 in the main text).

In Fig. A4 we display the eigenmode populations of the main eigenmodes in the steady-state response as a function of the incident light frequency. These populations correspond to the scattered light spectra shown in Fig. 4(b) of the main text. The peak of the subradiant mode excitation represents the case where the light scattering in the forward or back direction vanishes. In the limit of an infinitely large lattice with a subwavelength lattice spacing and fixed atomic positions only the exact forward or back scattering is possible, since in that case only the zeroth order Bragg peak survives.

The populations of the subradiant eigenmode excitation  $\mathcal{P}_P$  as a function of the Zeeman shifts are shown in Fig. A5 for both plane-wave and Gaussian incident fields. The maximum in the figure can be found using the effective two-mode model (see below). The full numerical calculation reveals high populations of the mode even for small values of  $\delta_\pm^z$ , as e.g., shown in the peak of the curve in Fig. A4. Around  $(\delta_+^z, \delta_-^z) = (0.05, 0.1)\gamma$  we find close to 95% excitation of the subradiant mode even for a plane-wave incident field and about 99% excitation for the Gaussian beam. Small values of  $\delta_\pm^z$  require notably longer evolution times before reaching the steady state. At small  $\delta_\pm^z$ , the large occupation values are also more sensitive to the precise field profile due to the edge effects of the lattice.

### Two-mode model

In the main section we introduced a phenomenological two-mode model in which case the two polarization den-

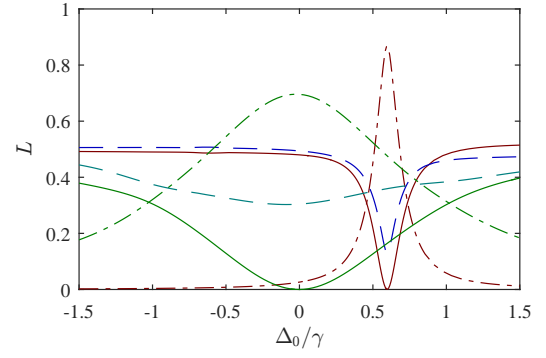


FIG. A4. The measure of the eigenmode populations  $L$  in the steady-state response as a function of the frequency of the incident light for fixed atomic positions. The occupations correspond to the spectrum of the forward or back scattered light shown in Fig. 4(b) of the main section.  $(\delta_+^z, \delta_-^z) = (0.1, 0.2)\gamma$ , sharp resonances: the collective subradiant eigenmode where the atomic dipoles are coherently oscillating in the  $x$  direction (red dash-dotted line), the collective eigenmode where the atomic dipoles are coherently oscillating in the  $y$  direction (red solid line), the sum of the mode populations of all the other 1198 eigenmodes (blue dashed line).  $(\delta_+^z, \delta_-^z) = (0.45, 1.75)\gamma$ , broad resonances: the same curves with green colors.

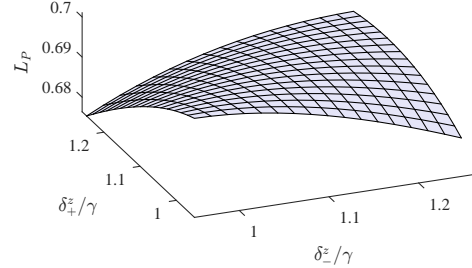


FIG. A5. The measure of the population  $L_P$  of the coherent subradiant eigenmode where the dipoles are pointing in the direction normal to the lattice in the steady-state response to a plane-wave excitation as a function of the Zeeman shifts;  $\Delta_0 = 0.65$ , with the fixed atomic positions. The excitation  $\mathcal{P}_P$  is maximized around  $\delta_+^z \simeq \delta_-^z$ ,  $\Delta_0 = -\delta_P$ .

sity amplitudes  $\mathcal{P}_P$  and  $\mathcal{P}_I$  obey the simplified dynamics given by

$$\dot{\mathcal{P}}_P = (i(\Delta_0 + \delta_P - \tilde{\delta}) - v_P)\mathcal{P}_P - \tilde{\delta}\mathcal{P}_I, \quad (\text{A1a})$$

$$\dot{\mathcal{P}}_I = (i(\Delta_0 + \delta_I - \tilde{\delta}) - v_I)\mathcal{P}_I + \tilde{\delta}\mathcal{P}_P + f, \quad (\text{A1b})$$

where  $\tilde{\delta} = (\delta_+^z - \delta_-^z)/2$ ,  $\bar{\delta} = (\delta_+^z + \delta_-^z)/2$  are defined in terms of the Zeeman shifts, and  $\Delta_0$  denotes the detuning of the  $m = 0$  state. The linear polarization of the incident field with the driving  $f = i\xi\epsilon_0\mathcal{E}_0/\mathcal{D}$  [ $\xi = 6\pi\gamma/k^3$ , with  $\gamma = \mathcal{D}^2k^3/(6\pi\hbar\epsilon_0)$ ] couples to the “coherent in-plane” collective eigenmode with the amplitude  $\mathcal{P}_I$  (resonance shift  $\delta_I$ , linewidth  $v_I$ ) in which all the atoms are coherently excited along the  $y$  direction. The “coherent perpendicular” eigenmode with the amplitude  $\mathcal{P}_P$  (resonance shift  $\delta_P$ , linewidth  $v_P$ ) represents a mode where all the atomic dipoles are oscillating in phase and pointing

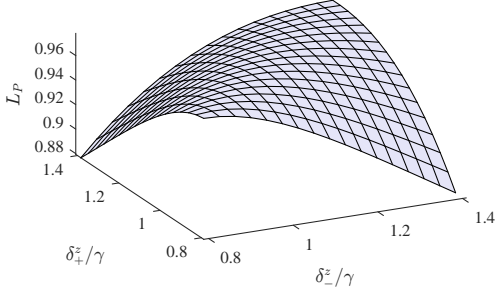


FIG. A6. The same as Fig. A5, but for a Gaussian beam excitation.

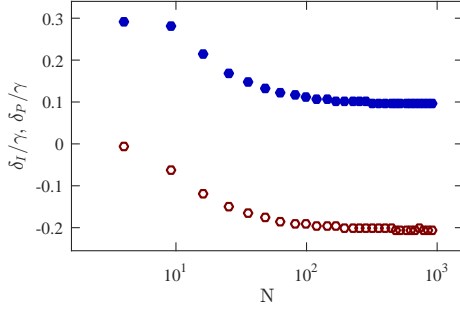


FIG. A7. The resonance shifts  $\delta_P$  (blue solid circles) and  $\delta_I$  (red rings) of the coherent perpendicular and in-plane collective modes, respectively, as a function of the atom number for fixed atomic positions with the lattice spacing  $a = 0.735\lambda$ . For the strongly subradiant collective mode with the dipoles coherently pointing normal to the plane, the resonance shifts become increasingly suppressed as the atom number increases.

normal to the plane. Example behavior of the resonance shifts of the two collective eigenmodes as a function of the lattice size are given in Fig. A7. In the limit of a large lattice the resonance shifts of the subradiant eigenmode become more suppressed.

The two-mode model qualitatively captures many of the essential features of the full many-body dynamics. This is illustrated in the spectra of Fig. 4 in the main text that shows the Fano resonances of the forward (or back) scattered light. In Fig. A8 we also show the comparison between the dynamics given by the two-mode model and the full numerics of all the 1200 collective excitation eigenmodes. For the plane-wave excitation the decay rates of the two cases differ at early times owing to the notable contribution of collective eigenmodes with different decay rates in Fig. A3(a). In the case of a Gaussian incident field excitation, the agreement is better, since in that case the entire excitation is dominated by a single collective eigenmode Fig. A3(c).

We may also easily calculate the steady-state solution

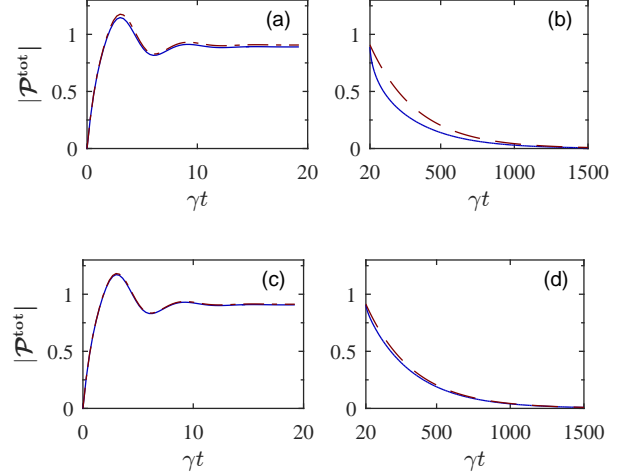


FIG. A8. Comparisons between the phenomenological two-mode model and the exact numerical simulation. The Zeeman shifts and the detuning  $(\delta_+^z, \delta_-^z, \Delta_0) = (1.1, 1.1, 0.65)\gamma$  have been chosen such that the collective polarization in the steady-state response is pointing normal to the lattice plane. In the two-mode model we have used the numerical values of the full eigenvalue calculation that are  $\delta_P = -0.65\gamma$ ,  $\delta_I = -0.68\gamma$ ,  $v_P = 0.0031\gamma$ ,  $v_I = 0.79\gamma$ . The initial evolution of the laser-driven lattice before reaching the steady-state distribution for the case of (a) plane-wave excitation, (c) Gaussian beam excitation. The evolution after the incident light and the Zeeman splittings are turned off for the case of (b) plane-wave excitation, (d) Gaussian beam excitation. In the case of a plane-wave excitation the two-mode model differs from the full numerical solution at early times due to the contribution of additional collective modes in the dynamics. The additional modes decay faster and the dynamics of the calculations is more similar at later times when the slowly decaying subradiant coherent perpendicular mode dominates.

of Eqs. (A1)

$$\mathcal{P}_I = -i \frac{Z_P(\Delta_0)}{\bar{\delta}^2 - Z_P(\Delta_0)Z_I(\Delta_0)} f \quad (\text{A2a})$$

$$\mathcal{P}_P = -i \frac{\bar{\delta}}{Z_P(\Delta_0)} \mathcal{P}_I, \quad (\text{A2b})$$

where we define

$$Z_{P/I}(\Delta_0) \equiv \Delta_0 + \delta_{P/I} - \bar{\delta} + iv_{P/I}. \quad (\text{A3})$$

The ratio of the amplitudes,

$$\left| \frac{\mathcal{P}_P}{\mathcal{P}_I} \right| = \left| \frac{\bar{\delta}}{Z_P(\Delta_0)} \right|, \quad (\text{A4})$$

indicates when the subradiant excitation  $\mathcal{P}_P$  becomes dominant. At the resonance  $\Delta_0 + \delta_P - \bar{\delta} = 0$  we have  $|\mathcal{P}_P/\mathcal{P}_I| = |\bar{\delta}|/v_P$ ,

$$|\mathcal{P}_P| = \left| \frac{\bar{\delta}}{\bar{\delta}^2 + v_I v_P} f \right|, \quad |\mathcal{P}_I| = \left| \frac{v_P}{\bar{\delta}^2 + v_I v_P} f \right|, \quad (\text{A5})$$

where we have assumed  $|\delta_P - \delta_I| \ll v_I$  so that we can neglect any difference between the  $\mathcal{P}_I$  and  $\mathcal{P}_P$  resonance frequencies. (We find that this holds approximately true for the lattice  $a \simeq 0.55\lambda$ .)

#### Forward and back scattered light

Since the dipoles of the  $\mathcal{P}_P$  excitation point in the direction normal to the plane, only the coherent in-plane collective mode can emit in the exact forward or back directions. Consequently, the forward and back scattered light amplitudes are in the steady-state response proportional to  $\mathcal{P}_I$  amplitudes, given by Eq. (A2a). We express the reflectance and the transmittance amplitudes in terms of the incident and the scattered field amplitudes  $\mathbf{E}_I$  and  $\mathbf{E}_S$ , respectively

$$r = \frac{\hat{\mathbf{d}} \cdot \mathbf{E}_S(-\hat{\mathbf{e}}_x)}{\hat{\mathbf{d}} \cdot \mathbf{E}_I(\hat{\mathbf{e}}_x)}, \quad t = \frac{\hat{\mathbf{d}} \cdot (\mathbf{E}_I(\hat{\mathbf{e}}_x) + \mathbf{E}_S(\hat{\mathbf{e}}_x))}{\hat{\mathbf{d}} \cdot \mathbf{E}_I(\hat{\mathbf{e}}_x)}. \quad (\text{A6})$$

We then obtain from Eq. (A2a)

$$r = \frac{r_0 v_I (v_P - i(\Delta_P - \tilde{\delta}))}{\tilde{\delta}^2 - (\Delta_P - \tilde{\delta} + i v_P)(\Delta_I - \tilde{\delta} + i v_I)}, \quad (\text{A7})$$

$$t = 1 + r, \quad (\text{A8})$$

where  $r_0$  denotes the reflectance amplitude at the resonance of the coherent in-plane collective mode when the Zeeman shifts vanish  $\tilde{\delta} = 0$ .

We can express the power reflectance  $|r|^2$  in the limit  $v_P/\tilde{\delta} \simeq 0$

$$|r|^2 \simeq \frac{(r_0 v_I)^2 [(\Delta_P - \tilde{\delta})^2 + v_P^2]}{|(\Delta_P - \tilde{\delta})^4 - (v_I^2 - 2\tilde{\delta}^2)(\Delta_P - \tilde{\delta})^2 + \tilde{\delta}^4|}, \quad (\text{A9})$$

where we have assumed  $|\delta_P - \delta_I| \ll v_I$ . In this limit we can then analytically calculate the half-width at half maximum for this resonance and obtain

$$w \simeq \frac{1}{2} \left( \sqrt{v_I^2 + 4\tilde{\delta}^2} - v_I \right). \quad (\text{A10})$$

This simple expression qualitatively explains the observed behavior of the spectral resonances of Fig. 4 in the main section. For the stronger driving of the  $\mathcal{P}_P$  excitation by the Zeeman shifts  $\tilde{\delta}$  the resonance is significantly broadened. In the limit of a weak driving, the resonance narrows and eventually only depends on the very narrow resonance linewidth  $v_P$ .

The scattering problem can be further simplified when we consider an infinite lattice on the  $yz$  plane for fixed atomic positions. The 2D lattice behaves similarly to a 2D diffraction grating. For the subwavelength lattice spacing only the zeroth order Bragg peak of the scattered light survives. This corresponds to the exact forward

and back scattered light. The energy conservation then states that  $|r|^2 + |t|^2 = 1$ . Combining this with Eqs. (A7) and (A8) yields  $r_0 = -1$ , indicating that an incident field at the resonance of the coherent in-plane collective mode experiences a total reflection when the Zeeman shifts are zero.

We can use this result to simplify the reflectance amplitude formula. We find a local minimum of the transmittance on  $\Delta_0 + \delta_P - \tilde{\delta} = 0$

$$r(-\delta_P + \tilde{\delta}) \approx -\frac{v_I v_P}{\tilde{\delta}^2 + v_I v_P} \quad (\text{A11})$$

where we have again assumed  $|\delta_P - \delta_I| \ll v_I$ . When the Zeeman shifts satisfy  $\tilde{\delta}^2 \gg v_P v_I$ , reflectance on  $\mathcal{P}_P$  resonance is suppressed, and transmittance is enhanced. This is illustrated in Fig. 4 of the main section where the resonances appear in the  $20 \times 20$  lattice. For the  $3 \times 3$  lattice the reflectance is only suppressed when  $(\delta_+^z, \delta_-^z) = (0.45, 1.75)\gamma$ , but not in the case of  $(\delta_+^z, \delta_-^z) = (0.1, 0.2)\gamma$  when  $\tilde{\delta}^2 \sim v_P v_I$ .

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